Azonosító jel:															

ÉRETTSÉGI VIZSGA ● 2012. május 8.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2012. május 8. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Important information

1. The exam is 240 minutes long, after that you should stop working.

2. You may proceed to solve the problems in arbitrary order.

3. In section II. you are required to solve only four out of the five problems. **Please remember to enter the number of the question you have not attempted into the empty square below.**
   Should there *arise any ambiguity* for the examiner about the question you ask not to be marked, it is question no. 9. that will not going to be assessed.

4. You may work with any kind of calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.

5. **Remember to show your reasonong when writing down the solutions; a major part of the score is given for this componenet of your work**

6. **Remember to include the substantial calculations in a clear manner.**

7. When refering to a theorem having a common name (e. g. Pythagoras' Theorem, sine rule) that you have done at school you are not expected to state it meticulously: it is usually sufficient to put he theorem's name. However, you are supposed to state clearly why and how does it apply. Any reference to any other theorem, however, is accepted only if it is stated precisely with all the conditions (no proof is needed) and you explain how it applies in the given situation.

8. Remember to answer each question (e. g. providing the result) also in text form.

9. You are supposed to work in pen; diagrams can still be drawn in pencil. Anything outside the diagram and written in pencil cannot be marked by the examiner. If a solution or some part of a solution is crossed out then it is not going to be marked.

10. There is only one solution of each problem to be marked. If you attempt a question more than once then you **should cleary indicate** the part you want to be marked.

11. Please, do not write anything in the shaded rectangular areas.

I.

1. The sides *a*, *b* and *c* of a given triangle satisfy the following equations:

$c = 2b$;
$a^2 + b^2 = 4$;
$a^2 - b^2 = 2$.

a) Find the lengths of the sides of this triangle.
b) Find the measure of the angles of this triangle.
c) Find the radius of the inscribed circle of this triangle.

You are expected to calculate the exact values of the respective results.

a)	4 points	
b)	5 points	
c)	4 points	
T.:	13 points	

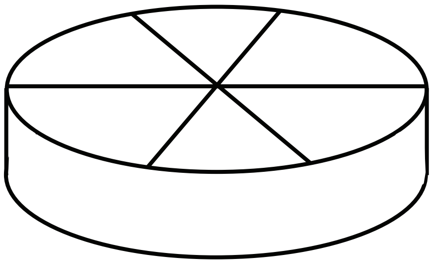
2.

a) A fair die is rolled twice and the two scores, in the order of their outcomes are entered for the digits a and b of the six digit number $\overline{8a567b}$, respectively. What is the probability that the digits of the six digit number hence obtained are all distinct?

b) Four sets are given as follows:
The elements of the set A are the two digit positive integers which are divisible by seven.
The elements of the set B are the two digit positive multiples of 29.
The elements of the set C are those two digit positive integers that are 11 less than a square number.
The elements of the set D are those two digit positive integers which yield a square number when decreased by 13.
b1) Find the number of elements of the set $A \cup C$.
b2) Find the number of elements of the set $B \cap D$.
b3) Find those two digit positive integers that belong to exactly two of the above four sets.

a)	4 points	
b)	8 points	
T.:	12 points	

3. Identical pieces of two types of cheese are packed into two cylindrical boxes. Six pieces that are labelled by red stickers are put in one of the boxes as shown on the diagram and another six that are labelled by blue stickers are put in the other box. The two times six identical pieces completely fill the respective boxes. These boxes are now emptied and the twelve pieces are put on a table. Six of them are put back into one of the boxes with their labels up. How many ways are there to arrange these six pieces in the box? (Two arrangements are different if neither of them can be obtained by rotating the other one.)



T.:	12 points	

4.

a) Given is the sequence $a_n = \frac{1}{7} \cdot \frac{1}{7^3} \cdot \frac{1}{7^5} \cdot \ldots \cdot \frac{1}{7^{2n-1}}$, $n \in \mathbf{N}^+$.

Find the greatest natural number n for which $a_n > 49^{-50}$.

b) Given is the sequence $b_n = \frac{1}{7} + \frac{1}{7^3} + \frac{1}{7^5} + \ldots + \frac{1}{7^{2n-1}}$, $n \in \mathbf{N}^+$.

Determine the limit $\lim\limits_{n \to \infty} b_n$.

a)	10 points	
b)	4 points	
T.:	14 points	

II.

You are expected to solve any four out of the problems 5 to 9. Write the number of the problem not selected in the blank square on page 3.

5.

a) The four vertices of a rectangle are given in the cartesian system as $A(0\,;0)$, $B(4\,;0)$, $C(4\,;1)$ and $D(0\,;1)$. An interior point $P\,(x\,;\,y)$ of the rectangle is selected randomly.

What is the probability that $y \le \frac{1}{3}x + \frac{1}{2}$?

b) Marci purchased 4 tombola tickets out of a total of 200 that were sold on the occasion of the Carnival. There were 10 prizes, altogether to be drawn. A ticket cannot win more than one prize.

b1) What is the probability that Marci wins exactly one prize on this tombola?
b2) What is the probability that Marci wins something on this tombola?

The calculations (the intermediate ones included) should be done correct to four decimal places.

a)	5 points	
b1)	5 points	
b2)	6 points	
T.:	16 points	

You are expected to solve any four out of the problems 5 to 9. Write the number of the problem not selected in the blank square on page 3.

6. The vertex of the graph of the quadratic function $f:\mathbf{R}\to\mathbf{R},\quad f(x)=ax^2+bx+c$ is $V(4;\ 2)$ and it is also given that the point $P(2;0)$ is lying on the graph.

a) Determine the coefficients a, b, and c.

b) Write down the equation of the tangent to the graph at its point whose abscissa is equal to 3.

c) Calculate the area bounded by the graph of f and the x-axis.

a)	6 points	
b)	5 points	
c)	5 points	
T.:	16 points	

You are expected to solve any four out of the problems 5 to 9. Write the number of the problem not selected in the blank square on page 3.

7. Solve the following equation on the set of real numbers.

$$6 \cdot \left(3^{\log_3 x}\right)^{\log_3 x} = \left(x^2\right)^{\log_3 x} - 6075 .$$

T.:	16 points	

You are expected to solve any four out of the problems 5 to 9. Write the number of the problem not selected in the blank square on page 3.

8. A company has three branches, one in each of three cities, respectively. The average ages of the employees of the Kőszeg branch, those of the Tata branch and those of the Füred branch are 37 years, 23, years and 41 years, respectively.

Three field trips were organized for the employees of the company. There were two branches going for each trip. There was no one else participating and each employee of the respective branches has, in fact, gone to the trip of their turn.

The first trip was organized for the Kőszeg and the Tata branches and the average age of the participants was 29. The average age of the participants of the second trip (those from the Kőszeg and Füred branches) was 39.5. Finally, the average age of the participants of the third trip – they came from the Tata and the Füred branches -- was 33.

What is the average age of the employees of this company?

T.:	16 points	

You are expected to solve any four out of the problems 5 to 9. Write the number of the problem not selected in the blank square on page 3.

9. An art gallery opened a new exhibition hall for children. The shape of the hall is a square based right pyramid whose internal dimensions are as follows: the edge of the base is 12 m long and the lateral edge is 10 m long. One of the exhibitor artists asked the contractors to stick a coloured band (a line) to hold the notices, all along the lateral walls, parallel to the edges of the base. The imaginary plane through the colored band exactly halved the volume of the exhibition hall.

a) Find the total length of the coloured band and also the height of the imaginary halving plane, above the ground level.

On the occasion of the opening ceremony the sound engineer hanged the microphone from the apex of the hall in such a way that it was equally distant from each lateral wall as well as from the ground.

b) Find the length of the hanging cable. You may neglect the size of the microphone and also that of the fastening.
(Give your answers corect to the nearest centimeter.)

a)	9 points	
b)	7 points	
T.:	16 points	

	number of the problem	maximal score	score attained	maximal score	score attained
Part I.	1.	13		**51**	
	2.	12			
	3.	12			
	4.	14			
Part II.		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Score on the written examination				**115**	

date

teacher

	rounded to the **next integer** (pontszám **egész számra** kerekítve)	**integer** score input for program (programba beírt **egész** pontszám)
Part I. (I. rész)		
Part II. (II. rész)		

teacher (javító tanár)

registrar (jegyző)

date (dátum)

date (dátum)